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Primordial Superstrings and the Origin of the Universe

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We discuss some of the cosmological implications of superstring theories. We pay particular attention to the issue of the initial state and its relationship with R-duality.

1. INTRODUCTION

Superstrings aim at being a consistent unified theory of the four known fundamental interactions in nature. They are currently the only candidate theory incorporating quantum gravitational effects which is not manifestly inconsistent [see Green *et al.* (1987) and Alvarez (1988*a*) for general reviews]. This means that we can address anew, at least in principle, the old questions that remain unsolved in classical cosmology, owing mainly to the existence of singularities in the spacetime itself, with the corresponding lack of predictability. These are questions such as the final outcome of the process of black hole evaporation, or the "physical meaning" of the initial state in cosmology, the "beginning" of the big bang.

Unfortunately, the state of the art in superstring theory does not allow for those nontrivial computations. Actually, although it is believed that spacetime itself is a derived concept, some sort of "condensate of strings," we only know how to compute amplitudes in a perturbative way around a given background (and even that is not clear for Riemann surfaces of arbitrary genus). This means that we lack all intuition on the physics of the very short distances (of the order of the Planck length) in which spacetime itself has probably "dissolved" and the appropriate dynamical framework should generalize the usual one consisting of a differentiable manifold [see Alvarez (1988*b*) for some speculations in this direction].

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In a more modest vein, however, there are ways of extracting useful information about the high-energy behavior of the theory. One of those (Gross, 1988) is based upon the study of the high-energy limit of the fixed-angle scattering amplitudes of superstrings. The result turns out to be surprisingly simple, and seems to imply the existence of an infinite number of linear relations between scattering amplitudes of different string states. Those relationships are valid order by order in topological perturbation theory, and have the generic form

$$A_{a_i}(p_i) = \prod_i V_{a_i}(x^{\mu}_{\mathrm{CL}}D, p_i) A_{\mathrm{TACH}}(p_i)$$

connecting the amplitude for scattering of string states a_i with momenta p_i with the corresponding amplitude for scattering of tachyons with the same external momenta, through the vertex operators V_{a_i}. The fact is that they connect amplitudes involving particles of different—and arbitrarily high—spins, which means that there are conserved charges with arbitrarily high spin, a fact forbidden by Haag, Lopuszanski, and Sohnius' theorem (an extension of the Coleman and Mandula's theorem allowing for supersymmetry). This is not as bad as it sounds, because there is one hypothesis in the theorem's proof which is not fulfilled in our situation. This is the particle finiteness hypothesis: we actually have an infinite number of massless particles in the $\alpha' \to \infty$ limit. The other alternative is, of course, that the theorem does hold in this case after all, and this implies that the S-matrix is trivial, which means a partonlike behavior of quasifree constituents in this limit. There are some reservations, though, on the exact meaning of the fixed-angle regime, and about the soundness of Goss and Mende's approximations [see, for example, Veneziano (1988)]. There seems to be agreement, however, on the general tendency toward softness of scattering amplitudes in this regime.

The other way of getting information on the physics of superstrings at very high energies is the study of superstrings at finite temperature [see Alvarez and Osorio (1989) for a general review].

The first thing to notice for thermal strings is that there is a critical temperature T_H such that, for $T > T_H$, the canonical partition function diverges. Actually, the asymptotic density of states for any string grows exponentially:

$$\omega(m) = \omega_0(m\sqrt{\alpha'})^{-a} \exp(bm\sqrt{\alpha'})$$

so that the partition function is

$$z \sim \int dm\, \omega(m)\, e^{-\beta E(m)} < \infty$$

only if $\beta > \beta_H = b\sqrt{\alpha'}$. There are now several possibilities (depending essentially on the value of the subdominant exponent a). When $z=\infty$, if $T > T_H$, but the free energy reaches a finite limit when the critical temperature is attained from below, i.e.,

$$\lim_{T\to T_H^-} F(T) < \infty$$

this is usually an indication that a phase transition should exist. This is exactly what happens in all theories involving closed strings only and, in particular, in the most interesting of them all, the heterotic string.

There is a close relationship between thermal strings and toroidal compactifications, and, when viewed in this form, the ultraviolet divergence at $T = T_H$ appears as an infrared one, due to the fact that a certain (winding) state becomes massless, with the corresponding appearance of a genus-zero contribution.

Atick and Witten (1989) claim that the high-temperature limit of superstrings is very soft; the free energy goes as T^2, whereas in ordinary quantum field theory it goes like T^{10} (in $d = 10$ dimensions). The limit is taken by keeping g^2T^2 constant in the case of superstrings, or g^2T constant in quantum field theory.

Another interesting point is the existence of β-duality (O'Brien and Tan, 1987; Alvarez and Osorio, 1988), which for heterotic strings amounts to

$$F_h(\beta) = (\pi^2/\beta^2)F_h(\pi^2/\beta)$$

Were this symmetry true, it would imply an even softer behavior of the free energy at high temperatures; actually, $\lim_{T\to\infty} F = 0$. There is no known physical theory with these properties. This is the main reason why it has been forcefully argued that β-duality must be broken, perhaps by a Wilson line in the timelike direction.

2. *R*-DUALITY

Let us now examine the $E_8 \times E_8$ heterotic string with d dimensions toroidally compactified. If the radii of the different circles are R^I ($I = 1, \ldots, \hat{d}$), and the torus $T^{\hat{d}} = \mathbb{R}^{\hat{d}}/\Lambda$, it is not difficult to check that the free energy is given by the expression

$$F_h = -L^d \prod_{I=1}^{\hat{d}} (2\pi R^I) \int_F \frac{d^2\tau}{\tau_2^2} \sum_{w,w'\in\Lambda} \exp\left[-2\pi\left(\tau_2 w^2 + \frac{1}{\tau_2}(w' - \tau_1 \cdot w)\right)\right]$$
$$\times\, \bar{\eta}^{12}\bar{\eta}^{-24}\bar{\theta}_{\Gamma_8+\Gamma_8}\tau_2^{-6}\left\{\theta\begin{pmatrix}0 & \frac{1}{2}\\ 0 & 0\end{pmatrix}\theta_2^4 + \theta\begin{pmatrix}\frac{1}{2} & 0\\ 0 & 0\end{pmatrix}\theta_4^4 - \theta\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ 0 & 0\end{pmatrix}\theta_3^4\right\}$$

where the Riemann theta functions (which carry all the dependence on the temperature) correspond to a matrix

$$\Omega = \frac{2i\beta^2}{\pi^2 \tau_2} \begin{pmatrix} |\tau|^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix}$$

and $L^d \to \infty$ is the volume of the uncompactified space.

It is quite easy to check, from direct inspection, that the free energy satisfies R-duality, that is,

$$F_h(R) = F_h(\alpha'/R)$$

This a very interesting symmetry indeed. Actually, it has been known for a long time that the spectrum and vacuum amplitudes of toroidally compactified strings obey R-duality. This result shows that thermal effects are not able to tell R and α'/R apart either. Several questions immediately come to mind. For example: to what extent is this symmetry generic, that is, valid for a "large" class of internal spaces? For the time being, this has been proved only for tori (with or without backgrounds) and for some orbifolds. The physical meaning of this symmetry (Brandenberger and Vafa, 1989) stems from the fact that the notion of position is a derived concept in toroidally compactified string theory. The usual definition is to perform a Fourier transform on momentum states, that is,

$$|x\rangle = \sum_p e^{ipx} |p\rangle$$

with the obvious periodicity

$$|x\rangle = |x + 2\pi R\rangle$$

But as the radius of the torus R goes down to zero, it becomes more and more costly (in energy) to create momentum states; simultaneously, the winding modes (which are forbiddingly energetic for large R) become correspondingly easy to create. This means that from the self-dual point $R^* = \sqrt{\alpha'}$ down, the "natural" definition of position states is

$$|\tilde{x}\rangle = \sum_w e^{iw\tilde{x}} |w\rangle$$

which have the periodicity

$$|\tilde{x}\rangle = |\tilde{x} + 2\pi/R\rangle$$

It is obvious that the preceding discussion depends in an essential way on the existence of winding modes, which in turn is a consequence of the internal manifold being nonsimply connected. It would thus seem that when the fundamental group $\pi_1 = 0$ (that is, when there are no winding modes, such as in all spheres S^n with $n > 2$) R-duality should disappear. More research should be done on this topic, though, and it might be that strings have some surprises in store concerning this problem, as they have with so many others. Another point is that, even if duality is an exact symmetry, it disappears in the thermodynamic limit. Let me belabor slightly this point [which is related to a discussion in Giveon *et al.* (1988)]. If we have an amplitude which is an analytic function of the symmetric variable $V+1/V$, that is,

$$A=\sum_{n=0}^{\infty} a_n(V+1/V)^n$$

in the thermodynamic limit $V\to\infty$ the amplitude reduces to

$$A=\sum_{n=0}^{\infty} a_n V^n$$

that is, R-duality is broken in this limit. This probably means that if it is a generic symmetry of strings one should compute everything at finite volume, taking care to never break this important symmetry. What is more exciting about this possibility is that, due to the fact that R-duality is a very "stringy" symmetry (we do not have winding modes for quantum fields), this is one of the few avenues in which observational consequences of the existence of strings could be expected.

3. THE ORIGIN OF THE UNIVERSE

In a very interesting paper, Brandenberger and Vafa (1989) [see also some related speculations in Alvarez (1985)] have proposed a scenario for the "spontaneous decompactification" of the four observed spacetime dimensions. They first observe that winding modes contribute negative pressure. This means that their energy grows linearly with the scale factor R, so that they would like to prevent expansion.

Of course what happens is that when they are in thermal equilibrium, there are not many of them around; they like to annihilate into momentum states (there are some global winding numbers conserved in these interactions, though). This means that, if they could, the winding modes would be at equilibrium with the momentum states, in such a way that the total number of winding modes never gets too high. But in order for this to be possible at all, winding modes have to interact, and this is only possible if $d\leq 4$ (this is because strings will generically miss each other if $d>1$, due to the famous theorem that $2+2=4$). Of course, this does not explain why $d<4$ is not favored, but at first sight looks like a promising first step in order to gain some understanding into the physical problem at hand. There are some deep problems with this scenario, though. In spite of the fact that

Einstein's equations are not valid for superstrings, let us use them in order to get some intuition. For a Friedmann-Robertson-Walker universe, they read (e.g., Dominguez-Tenreiro and Quiros, 1988)

$$3\ddot{R} = -4\pi G(\rho+3p)R+\Lambda R$$

$$R\ddot{R}+2\dot{R}^2+2\kappa = 4\pi G(\rho-p)R^2+\Lambda R^2$$

where the constant $\kappa = \pm 1$, 0, according to the character of the constant-curvature 3-surfaces $t = \text{const}$. These two equations together imply Friedmann's "energy equation"

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi g}{3}\rho + \frac{\Lambda}{3} - \frac{\kappa}{R^2}$$

On the other hand, for a perfect fluid

$$T_{\mu\nu} = (\rho+p)u_\mu u_\nu - pg_{\mu\nu}$$

and the covariant conservation law $\nabla T = 0$ implies

$$\frac{d}{dR}(\rho R^3) = -3pR^2$$

We first observe that a cosmological constant corresponds to negative pressure, $p = -\rho$. When $\Lambda > 0$, we immediately deduce from equation (3.2) that the expansion is favored (this is actually what happens in an inflationary phase of the expanding universe). To the extent that Einstein's equations are a good guide to our intuition, then, it is not clear that winding modes must necessarily prevent expansion. Another thing is that, even if winding states do prevent inflation after all, it has been argued several times [see, for example, Gross (1984) and the general discussion in Green *et al.* (1987)] that the Hausdorff dimension d_H of strings should be infinite. Although this has been criticized by Frölich and collaborators, it is clear that strings will interact in spacetimes with dimensions bigger than 4, which means that they will probably reach equilibrium and disappear dynamically. In conclusion, more work is needed before an intrinsically stringy cosmological scenario appears. One of the more urgent problems is to gain some intuition on the short-distance regime, in which the approximation of the existence of a spacetime continuum is not a good one, and the best description we can make of our string theory is a discrete set of two-dimensional conformal fields. Perhaps as a substitute for spacetime there should be a general metric space, as has been proposed in Alvarez (1988*b*), but most likely, more radical ideas are needed. Difficult as it will be, this is one of the most fascinating problems for the human mind to ponder: the reward is to get a new physical picture of the origin of the universe itself.

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REFERENCES

Alvarez, E. (1985). *Physical Review D*, **3**, 418.

Alvarez, E. (1989). Quantum gravity: An introduction to some recent results, *Rev. Mod. Phys.*, **61**, 561.

Alvarez, E. (1988*b*). *Physics Letters*, **210B**, 73.

Alvarez, E., and Osorio, M. A. R. (1988). *Nuclear Physics B*, **304**, 327.

Alvarez, E., and Osorio, M. A. R. (1989). Thermal heteroric strings, *Physica A*, **158**, 449.

Atick, J., and Witten, E. (1989). Preprint, Princeton University.

Brandenberger, R., and Vafa, C. (1989). Preprint, Brown University.

Dominguez-Tenreiro, R., and Quiros, M. (1988). *An Introduction to Cosmology and Particle Physics*, World Scientific, Singapore.

Giveon, A., Rabinovici, E., and Veneziano, G. (1988). Preprint CERN TH 5106/88.

Green, M. B., Schwarz, J., and Witten, E. (1987). *Superstring Theory*, Cambridge University Press, Cambridge.

Gross, D. (1984). *Physics Letters*, **138B**, 185.

Gross, D. (1988). *Physical Review Letters*, **60**, 1229.

O'Brien, R., and Tan, C. (1987). *Physical Review D*, **36**, 1175.

Veneziano, G. (1988). Preprint CERN-TH88.